ARKIV FÖR FYSIK Band 2 nr 4

Read 7 December 1949

The mode of operation of the uranium reactor

By Ragnar Liljeblad

An article dealing with this subject and written by the present author has already been published.[1] It was produced in view of the possible future use of the uranium reactor as a source of energy, and with the aim of estimating the quantity of energy that might be obtained from such a reactor under various suppositions. The most significant conclusions may be summarized in the following points.

Either it is possible to employ a design of pile which enables energy to be extracted also from U 238, as the quantity of fissionable matter will not diminish with time, *or else* it is only possible to extract a quantity of energy of the same order as that available in U 235. A third alternative — which excludes the possibility of the first, — for instance, extracting 5 to 10 times the amount of energy available in U 235, is not possible to realize. In other words: If the last mentioned alternative should present itself, it would be equally possible to construct a pile operating according to the first method, in which case that would naturally be the one chosen. Should alternative one not be feasible, then alternative three cannot be realized either, and only the second alternative remains.

The above investigation was based, however, on assumptions which are only true under certain suppositions. There is especially one statement in Smyth's report 4–6 (b) on which I have based my deductions, namely that the number of neutrons emitted for each thermal neutron absorbed by the uranium is only somewhat smaller than the number of neutrons emitted for each nuclear fission. According to this, the thermal absorption by U 238 may be neglected, and it follows that practically all absorption by U 238 consists of resonance absorption of comparatively high-speed neutrons.

According to other reports it would seem, however, that the number of thermal neutrons absorbed by U 238 is not negligible in comparison with those absorbed by U 235, in any case, not in a reactor of natural uranium which is not enriched. In the following I have therefore given a more complete account, in which consideration has also been taken of the thermal absorption by U 238. Nevertheless, I find that the conclusions arrived at in the previous article still hold good.

---

[1] Arkiv för matem. etc. Bd 36 B. N:o 1 (Article submitted in March 1948).

It is evident that if for each split nucleus so many secondary neutrons are liberated that, after deduction of those absorbed by the moderator or foreign bodies, or by necessary constructional details of the reactor, there are two effective neutrons remaining even after the reactor has been adjusted to a multiplication factor of unity by means of special control rods made of cadmium, then the rate at which fresh Pu 239 nuclei are formed will balance the rate at which U 235 nuclei or previously formed Pu 239 nuclei disintegrate. The quantity of U 235 + Pu 239 nuclei will therefore remain constant and the reaction can proceed until practically all the U 238 has been consumed. Although this may not be an accurate representation, it nevertheless approximates to actual conditions. In reality the quantity of U 238 diminishes with time as, however large it may be in relation to the quantity of U 235, it is not unlimited. When the transformation has proceeded so far that there is a noticeable diminution in the quantity of U 238, the rate at which fresh plutonium is generated will also diminish. The reactor should therefore be designed from the very beginning to produce a fraction of 1 per cent excess of fresh plutonium, so that the quantity of fissionable matter can be transformed once or twice without any diminution in the quantity. When the quantity ultimately begins to diminish the reactor must be stopped and the neutron-absorbing fission products are removed, while the uranium with the enclosed plutonium is recast together with a fresh charge of uranium, or pure U 238 if available, or thorium, to a quantity which corresponds to the consumed quantity of U 238. In this manner it is possible to retain the reactor in service indefinitely, and all the U 238 or thorium will be consumed. It is selfevident that right from the beginning one should aim also at a multiplication factor slightly in excess of unity to provide for an absorption by the control rods and thus avoid the need for taking the reactor out of service too soon for the purpose of removing harmful fission products.

However, what would be the position if the number of remaining active neutrons resulting from each nuclear fission should average between 1 and 2 (if below unity, it is evident that no chain reaction can be initiated).

First of all, it is obvious that if the reactor is adjusted to a multiplication factor of 1 and is then left untouched, the reaction will gradually cease, even if we assume that the harmful fission products are removed. Let us assume, for instance, that the quantity of fissionable nuclei has been halved. A neutron which is emitted when a nucleus disintegrates, and which after being retarded to thermal velocity is intended to disintegrate another nucleus, will then have to travel around for twice as long before striking a fresh fissionable nucleus. Such will anyway be the case if the metal and moderator form a homogeneous mixture. With a reactor of normal design, the above will only be an approximation to the actual conditions, but it becomes increasingly valid if the uranium rods consist of comparatively thin laminations, such as will in any case be necessary in order to ensure effective cooling. The chances of the neutron being absorbed by U 238, the cooling medium, the moderator etc., are therefore doubled. If the reactor is to remain in service so long that the quantity of fissionable nuclei diminishes to half when the cadmium rods are fully withdrawn, the initial position of the latter must be adjusted to absorb about the same quantity of neutrons as the remaining absorbing media in the reactor. By fully withdrawing the rods it will be possible to reduce the absorption by about half and a thermal neutron may therefore travel for twice as long before being absorbed.

It should be noted that that fraction of the emitted secondary neutrons, which is initially absorbed by resonance by the U 238, does not diminish noticeably, as the quantity of U 238 does not alter appreciably and the harmful absorption is almost entirely confined to the thermal regions (see Smyth's report 8–11) and thus does not affect the neutrons in the resonance region, around 10 eV.

In the following the number of neutrons liberated by each nuclear fission will be denoted a, and the multiplication factor of a reactor with the control rods fully withdrawn will be denoted $1 + \varepsilon$. It is quite evident that the reactor having a multiplication factor greater than unity cannot remain in operation for more than a second or so, and in actual service the excess must be absorbed by the cadmium rods. When the reactor is new $\varepsilon = \varepsilon_0$. Furthermore, we assume that for each nuclear fission of a neutrons, μ_r will be absorbed in the resonance region by the U 238 and will form plutonium. As already pointed out, μ_r will depend upon the design of the reactor and does not change appreciably during the time the reactor is in service. The thermal absorption by the U 238 when the multiplication factor is 1 with the control rods fully withdrawn is denoted μ, while the initial value, at the multiplication factor of $1 + \varepsilon_0$, is written μ_0. The corresponding value of the harmful absorption in the moderator and other parts is denoted ν and ν_0 respectively. N represents the quantity of fissionable nuclei per cm^3 in a reactor which has just ceased functioning because the multiplication factor has fallen to unity, and N_0 the fissionable nuclei in a new reactor with multiplication factor $1 + \varepsilon_0$. The fission cross section for U 235 is denoted σ_f (that for Pu 239 is assumed to be equally as large as for U 235), and the capture cross section for U 238 is denoted σ_c and for absorption in the moderator σ_m; the number of nuclei per cm^3 of the moderator is referred to as N_m. To begin with we assume the existence of a large, homogenous reactor in which the uranium metal and the moderator are homogenously mixed.

The following equations then apply:

$$N_0 - N = s N_0 - (\mu_r + \mu) s N_0$$

$$s = \frac{1}{1 - \mu_r - \mu} \left[1 - \frac{N}{N_0} \right] \tag{1}$$

and

$$a = 1 + \mu_r + \mu + \nu = 1 + \varepsilon_0 + \mu_r + \mu_0 + \nu_0 \tag{2}$$

$$\mu = \frac{\sigma_c \cdot 140\, N_0}{\sigma_c \cdot 140\, N_0 + \sigma_f N + \sigma_m N_m} (a - \mu_r)$$

or

$$\mu = \frac{140\, \sigma_c}{140\, \sigma_c + \sigma_f \dfrac{N}{N_0} + \sigma_m \dfrac{N_m}{N_0}} (a - \mu_r) \tag{3}$$

$$\nu = \frac{\sigma_m \dfrac{N_m}{N_0}}{140\,\sigma_c + \sigma_f \dfrac{N}{N_0} + \sigma_m \dfrac{N_m}{N_0}} (a - \mu_r) \tag{4}$$

$$1 = \frac{\sigma_f \dfrac{N}{N_0}}{140\,\sigma_c + \sigma_f \dfrac{N}{N_0} + \sigma_m \dfrac{N_m}{N_0}} (a - \mu_r). \tag{5}$$

If we put

$$\nu_0 = \frac{\sigma_m \dfrac{N_m}{N_0}}{140\,\sigma_c + \sigma_f + \sigma_m \dfrac{N_m}{N_0}} (a - \mu_r) = k\,(a - \mu_r) \tag{6}$$

we obtain

$$\frac{1}{k} = \frac{140\,\sigma_c + \sigma_f + \sigma_m \dfrac{N_m}{N_0}}{\sigma_m \dfrac{N_m}{N_0}}$$

$$\sigma_m \frac{N_m}{N_0} = \frac{140\,\sigma_c + \sigma_f}{\left(\dfrac{1}{k} - 1\right)}. \tag{7}$$

According to available information $\sigma_f \simeq 455 \cdot 10^{-24}$ cm^2 and $\sigma_c \simeq 2.6 \cdot 10^{-24}$ cm^2. From this we get

$$\sigma_m \frac{N_m}{N_0} = \frac{819}{\left(\dfrac{1}{k} - 1\right)} \cdot 10^{-24}. \tag{7 a}$$

By inserting this value and values of σ_c and σ_f in equations (3) and (4) and simultaneously dividing the equations by equation (5) in order to simplify the expressions, we obtain

$$\mu = 0.8 \frac{N_0}{N} \tag{3 a}$$

and

$$\nu = \frac{1.8}{\left(\dfrac{1}{k} - 1\right)} \frac{N_0}{N} \tag{4 a}$$

and

$$\frac{455\,\dfrac{N}{N_0}}{364+\dfrac{819}{\left(\dfrac{1}{k}-1\right)}+455\,\dfrac{N}{N_0}}\,(a-\mu_r)=1 \tag{5 a}$$

from which

$$\frac{N}{N_0}=\frac{3.64+\dfrac{8.19}{\left(\dfrac{1}{k}-1\right)}}{4.55\,(a-1-\mu_r)}. \tag{8}$$

$\frac{N}{N_0}$ and s are then calculated for various values of k and μ_r. According to the latest information a is somewhat less than 2.6, and in the following we shall employ the value $a = 2.60$, whereby the so called "fast fission effect" is taken into account by incorporating in a a factor $\simeq 1.03$.

I. $k = 0.25$

$$\mu_r = 0 \qquad \frac{N}{N_0} = 0.875 \qquad s = 1.47$$

$$\mu_r = 0.1 \qquad \frac{N}{N_0} = 0.934 \qquad s = 1.54$$

$$\mu_r = 0.2 \qquad \frac{N}{N_0} = 1 \qquad s = 1.66 \quad [\mu_r + \mu = 1]$$

II. $k = 0.27$

$$\mu_r = 0 \qquad \frac{N}{N_0} = 0.92 \qquad s = 0.62$$

$$\mu_r = 0.1 \qquad \frac{N}{N_0} = 0.98 \qquad s = 0.24$$

$$\mu_r = 0.13 \qquad \frac{N}{N_0} = 1 \qquad s = 0$$

III. $k = 0.30$

$$\mu_r = 0 \qquad \frac{N}{N_0} = 0.98 \qquad s = 0.11$$

$$\mu_r = 0.03 \qquad \frac{N}{N_0} = 1 \qquad s = 0.$$

It is interesting to note that if the harmful absorptions are so small that the reactor may be designed to operate immediately below the breeding limit, it should be dimensioned to give a μ_r value as large as possible, up to 0.20. (Should the breeding limit be exceeded, μ_r may evidently assume even higher values). On the other hand, if the harmful absorption is only slightly greater, the reactor should be designed for the lowest possible resonance absorption in order to ensure a maximum of s.

The possibilities of varying μ_r in a homogeneous reactor are likely to be comparatively limited (but not so in a lattice reactor, i.e. a reactor in which the uranium rods are located in a lattice pattern in a large moderator mass). Increasing the uranium at the expense of the moderator mass must anyhow result in an increase in μ_r. It should be noted that in the event of an alteration in this direction, k cannot be regarded as altogether constant if the identical type of moderator mass is employed. The above comparisons of different μ_r values, but with the identical k value, presupposes therefore a certain minor change in the moderator quality. In reality ν_0 diminishes at a slightly greater rate than the reduction in the number of thermal neutrons $(a - \mu_r)$ brought about by a comparative reduction in the moderator mass. (See also equation (6)). As will be seen from the previous article, however, this condition, if taken into account, would only serve to accentuate the discontinuity at the breeding limit, which the introduction of the linear change of ν_0 with $(a - \mu_r)$ has already emphasized.

The above account should give a fairly true picture of the actual conditions in the large homogenous reactor.

However, it is also a matter of considerable interest to investigate how conditions change in a lattice reactor, even in one of limited size. Here conditions are a good deal more complicated, and it is not feasible to carry out an accurate calculation with the data available at present. Nevertheless, it is possible to calculate the theoretically ideal case, giving the highest possible energy output. This will at least serve to tell us that the actual conditions will be less favourable than in the hypothetical case chosen. We assume therefore that all resonance absorption takes place in the uranium rods at which the neutrons in reality are absorbed but that absorption of all thermal neutrons, which will gradually be absorbed by the metal, takes place the first time they strike a uranium rod after travelling around in the moderator mass for some time. The harmful absorption, including the loss by escaped neutrons, may then be regarded as constant and we may put $\nu = \nu_0$. ν is the harmful absorption when the reactor has just ceased operating because the multiplication factor has fallen to unity with fully withdrawn control rods, and ν_0 represents the harmful absorption in a new reactor, also with the control rods withdrawn, and consequently having a multiplication factor of $1 + \varepsilon_0$. (The fact that the reactor is not able to operate so for more than about a second obviously does not affect the argument). The difference in the reactor in its initial and final stages is only that, of the thermal neutrons which are absorbed by the uranium, a greater portion is absorbed in the U 238 in the final stage, causing the multiplication factor to fall from $1 + \varepsilon_0$ to 1. In the moderator nothing is changed.

It is also evident that this hypothetical case is more favourable than the reactor would be in reality. In actual practice a number of the thermal neutrons would have to pass through the metal without being absorbed by the

U 235 or Pu, and the smaller the quantity of fissionable nuclei, the more often would this occur. Neutrons would therefore travel about for a longer time in the moderator before being absorbed by U 235 or Pu and the harmful absorption as well as the useful absorption in U 238 would increase.

The assumption that k is a constant, i.e. that the redesign of the reactor for the purpose of obtaining a greater resonance absorption μ_r will lead to a reduction of the absorption in direct proportion to the number of thermal neutrons $(a - \mu_r)$, should approximate closely to the truth in the case of a lattice reactor, as redimensioning it in order to increase μ_r does not necessarily imply that the relation between the moderator mass and the mass of metal is reduced, but only that the uranium rods will be thicker, but fewer in number owing to their wider spacing. If k should possibly diminish somewhat with $(a - \mu_r)$, as in the case of the homogeneous reactor, this would only imply that the discontinuity at the breeding limit becomes more emphatic.[1]

Equations (1), (2) and (6) therefore still apply

$$s = \frac{1}{1 - \mu_r - \mu}\left[1 - \frac{N}{N_0}\right]$$

and

$$a = 1 + \mu_r + \mu + \nu = 1 + \varepsilon_0 + \mu_r + \mu_0 + \nu_0$$

$$\nu_0 = k\,(a - \mu_r)\,.$$

By putting $\nu = \nu_0$ we get

$$\mu = (a - \mu_r)\,(1 - k) - 1\,. \tag{9}$$

Evidently, the following still applies

$$\mu = \frac{140\ \sigma_c}{\sigma_f}\,\frac{N_0}{N}\,. \tag{10}$$

---

[1] It may at first sight appear confusing to say that the assumption that k equals a constant gives a somewhat excessive liberation of energy at the breeding limit. It might seem that if redesign of the reactor results in the harmful absorption diminishing, or diminishing more than expected, this would have the very opposite effect. However, such is not the case. The question should be approached in the following manner: — Assume that the reactor has just ceased to function because the multiplication factor has fallen to a value immediately below unity. We also assume that this reactor is so designed, and constructed of such material, that redesigning the uranium rods and the lattice pattern only just enables it to operate infinitely close to the breeding limit, at which the liberation of energy is at a maximum. This means that the useful absorption $\mu_r + \mu$ must increase by a certain amount in order to attain the value 1. In order to facilitate this increase we have firstly ε_0 neutrons, i.e. the quantity of neutrons by which the multiplication factor exceeded unity in the original reactor. But there is also the reduction achieved in the harmful absorption by the expedient of redesigning the reactor. On the assumption that the reactor in practice can only just be redesigned so as to operate on the breeding limit this will mean that due to the diminution in the harmful absorption, ε_0 is not so large as would otherwise be required in order to facilitate the redesign. For this reason the reactor cannot have consumed as many generations of fissionable nuclei or have developed as much energy as would have been the case if ε_0 had been larger. This holds true however close to the breeding limit the reactor may have operated from the beginning. Moreover, this condition may readily be deduced mathematically from the previous calculations.

Increasing the previously assumed values of σ_f and σ_c, we get

$$\mu = 0.8 \frac{N_0}{N}.$$

From (9) and (10 a) we obtain

$$\frac{N}{N_0} = \frac{0.8}{(a - \mu_r)(1 - k) - 1} \tag{11}$$

and

$$s = \frac{1}{2 - a(1 - k) - k\,\mu_r}\left[1 - \frac{0.8}{(a - \mu_r)(1 - k) - 1}\right]. \tag{12}$$

Assuming different values for k and μ_r

$$I. \quad k = 0.25$$

$$\mu_r = 0 \qquad \frac{N}{N_0} = 0.84 \qquad s = 3.2$$

$$\mu_r = 0.1 \qquad \frac{N}{N_0} = 0.91 \qquad s = 3.4$$

$$\mu_r = 0.2 \qquad \frac{N}{N_0} = 1 \qquad s = 3.75 \quad [\mu_r + \mu = 1]$$

$$II. \quad k = 0.27$$

$$\mu_r = 0 \qquad \frac{N}{N_0} = 0.89 \qquad s = 1.10$$

$$\mu_r = 0.1 \qquad \frac{N}{N_0} = 0.97 \qquad s = 0.41$$

$$\mu_r = 0.13 \qquad \frac{N}{N_0} = 1 \qquad s = 0$$

$$III. \quad k = 0.30$$

$$\mu_r = 0 \qquad \frac{N}{N_0} = 0.975 \qquad s = 0.14$$

$$\mu_r = 0.03 \qquad \frac{N}{N_0} = 1 \qquad s = 0.$$

It will be seen therefore that the previously mentioned discontinuity at the breeding limit will occur also in a lattice reactor, although it will not be so marked. Nevertheless, it should be noted that the actual output will be somewhere between the above values and the value for a homogeneous reactor.

It is also interesting to compare a lattice reactor with a homogeneous reactor, and evidently the former will have a substantially greater output, although this is true only while it operates in the neighbourhood of the breeding limit. Immediately the harmful absorption increases by even an insignificant amount, the energy output will fall off considerably, and the difference in output between a lattice reactor and a homogeneous reactor will not be great. It will be seen that the greatest value of μ_r at which a new reactor is unable to operate because the multiplication factor barely reaches unity will be the same whatever the design of the reactor, if the coefficient k for the harmful absorption is the same in every case. This also becomes immediately obvious if one considers that, of the liberated neutrons, 1.8 thermal neutrons must be absorbed in the normal uranium metal at a multiplication factor of 1, and for the same value of μ_r and k there will evidently be the same number of thermal neutrons remaining for the uranium metal irrespective of the reactor design. The choice of a lattice reactor in preference to a homogeneous reactor (specially when working below the breeding limit) is therefore mainly based on the fact that in a lattice reactor it is easier to attain a lower value of μ_r and so to bring the multiplication factor up to unity. Besides, the cooling possibilities are better.

It is also interesting to note that if a new reactor is able to operate at all, though μ_r is made as small as possible, it is possible to rebuild it for breeding, if the harmful absorption may be decreased by 17 % ($k = 0.25$ instead of $k = 0.30$).

Finally, it should be mentioned that the effect of neutron-absorbing fission products has been disregarded in the above. As it is not possible to remove such products continuously, but only on certain occasions when the reactor possibly must be stopped, the values of s quoted above must be regarded as theoretically ideal maximum values which could only be attained if no fissionable matter were lost in the course of repeated purifying of the reactor. It should also be noted that the entire analysis is based on the assumption that in the case of a reactor operating below the breeding limit — and also, as previously mentioned, in the case of a reactor operating beyond the breeding limit — the consumed quantity of U 238 is replenished, or replaced by thorium, when the reactor is stopped for the purpose of removing fission products. However, this supposition is relatively unimportant as regards a reactor operating below the breeding limit, as s is invariably small and even in the most extreme case the diminution in the quantity of U 238 is insignificant.

Tryckt den 28 februari 1950

Uppsala 1950. Almqvist & Wiksells Boktryckeri AB